ought to be disregarded, but they do not forward any criterion that might be useful for recognizing, i.e. identifying, such data. Hence, the new method owes its simplicity to an incorrect elimination of the effect of side diffusion.

Moreover, problems arising due to the necessity of controlling the temperature of a sizeable bulk of liquid should be considered; especially near the surface the layer being measured may be cooled by evaporation to the overlying gas which is not saturated by the liquid vapour. With large volumes of liquid it also becomes more difficult to prevent mechanically induced convective fluxes and their fluctuations. Hence, the new method seems to be rather vulnerable because of serious doubts about its usefulness, in spite of the fact that it has already been popularized in at least four publications (Ju and Ho, 1985, 1986; Ho *et al.*, 1986; Ju *et al.*, 1986). According to Ju and Ho (1986) "the literature appears to be scattered with information which is confusing and often mutually contradictory. . . It appears that the controversy in large might be related to the problems associated with different experimental techniques." But this criticism fits their own work to perfection.

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Comments on improvements on a replacement for the logarithmic mean

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Dear sirs,

In a recent publication Paterson [1] gave an approximation to the logarithmic mean temperature difference (LMTD). In the range of interest for heat exchanger design, it gave close approximations to the LMTD and it also has the advantage that it approaches the correct limit as the two temperature differences approach one another. Furthermore, it is mathematically tractable and provides an explicit solution when the outlet temperature and the flow rate of one stream, say the cooling stream, are the unknowns.

Paterson's approximation θ_{PM} to the LMTD is given as

$$\theta_{LM} \simeq \theta_{PM} = \tfrac{2}{3}\theta_{GM} + \tfrac{1}{3}\theta_{AM} \tag{1}$$

where

$$\theta_{GM} = (\theta_1\theta_2)^{1/2} \tag{2}$$

$$\theta_{AM} = \tfrac{1}{2}(\theta_1 + \theta_2). \tag{3}$$

Equation (1) may be substituted into the rating equation of a heat exchanger

$$Q = UA\theta_{LM} \tag{4}$$

which for given (Q/UA) may be simplified to result in a quadratic involving θ_1 and θ_2 and, after some rather laborious manipulation, θ_1 is obtained as an explicit solution of θ_2

$$\theta_1 = \left(7\theta_2 + 6\frac{Q}{UA}\right) - 4\sqrt{3}\left(\theta_2^2 + 2\theta_2\frac{Q}{UA}\right)^{1/2}. \tag{5}$$

In the counter-current heat exchanger example shown by Paterson [1], the exit temperature may be calculated from θ_1, as all the terms on the RHS of eq. (5) are known.

It is interesting to note that the mean temperature difference approximation given by Paterson [1] may be viewed as a weighted arithmetic mean of θ_{GM} and θ_{AM}. One could also derive an approximation based on the weighted geometric mean of θ_{GM} and θ_{AM}

$$\theta_{CM1} = \theta_{GM}^a\theta_{AM}^b \tag{6}$$

where $a + b = 1.0$, and it would simplify the subsequent analysis if $a = 2b$.

Thus, one could try

$$\theta_{CM1} = \theta_{GM}^{2/3}\theta_{AM}^{1/3} \tag{7}$$

which will result in, after simplification,

$$\theta_{CM1} = \left(\frac{\theta_1\theta_2^2 + \theta_1^2\theta_2}{2}\right)^{1/3}. \tag{8}$$

If eq. (8) is substituted into eq. (4), θ_1 may again be obtained as an explicit function of θ_2:

$$\theta_1 = -\frac{\theta_2}{2} + \sqrt{\left[\frac{\theta_2^2}{4} + \frac{2}{\theta_2}\left(\frac{Q}{UA}\right)^3\right]}. \tag{9}$$

The derivation and the form of eq. (9) are much simpler than for the case of eq. (5) given by Paterson [1].

The two approximations given above are based on the consideration of some weighted mean values of θ_{GM} and θ_{AM}.

A search in the literature revealed that the celebrated British chemical engineer, A. J. V. Underwood [2, 3], had also given an approximation to θ_{LM}, in 1933, based directly on θ_1 and θ_2:

$$\theta_{UM}^{1/3} = \tfrac{1}{2}(\theta_1^{1/3} + \theta_2^{1/3}). \tag{10}$$

While Underwood [3] probably derived eq. (10) with the aid of a slide rule, it is possible to fine-tune the exponential index to obtain a better match with θ_{LM}. This results in

$$\theta_{CM2}^{0.3275} = \tfrac{1}{2}(\theta_1^{0.3275} + \theta_2^{0.3275}). \tag{11}$$

When either eq. (10) or (11) is applied to eq. (4) and solving for θ_1 in terms of θ_2 the following result is simply obtained:

$$\theta_1 = \left[2\left(\frac{Q}{UA}\right)^n - \theta_2^n\right]^{1/n} \tag{12}$$

where $n = \frac{1}{3}$ or 0.3275 depending on whether eq. (10) or (11) was used.

To compare the accuracies of the predictions of Paterson's approximation and those given in this work, θ_{LM}, θ_{PM}, θ_{CM1}, θ_{UM} and θ_{CM2} are evaluated and given in Table 1 for the same θ_1 and θ_2 values used by Paterson [1].

With reference to Table 1, it is observed that while eq. (8) provides a simpler solution than eq. (1), its accuracy is slightly inferior. However, the approximation given by Underwood in 1933 which results in a very simple solution as evidenced by eq. (12) also gives superior approximations to θ_{LM} compared to that proposed by Paterson [1]. Furthermore, by slightly adjusting the exponential index to give θ_{CM2} as modified in this work, the approximation gives an essentially exact match with θ_{LM} in the range of θ_1 and θ_2 values considered. The application of eq. (12) to the worked example given by Paterson [1] is straightforward and need not be given here.

Table 1. Comparison of the various approximations with θ_m, the logarithmic mean

θ_1	10	10	10	10
θ_2	15	20	50	100
θ_{LM}	12.33	14.43	24.85	39.09
θ_{PM} [eq. (1)]	12.33	14.42	24.91	39.42
θ_{CM1} [eq. (8)]	12.33	14.42	24.66	38.03
θ_{UM} [eq. (10)]	12.33	14.42	24.88	39.24
θ_{CM2} [eq. (11)]	12.33	14.42	24.84	39.09

CONCLUSION

A slight modification of the approximation to θ_{LM} given by Paterson [1] results in a simpler solution although its accuracy is slightly inferior. An approximation given by Underwood in 1933 [2, 3] was shown to give not only a very simple solution but also superior results. Furthermore, a slight modification to the Underwood approximation results in eq. (11) giving θ_{CM2} which is in almost exact agreement with θ_{LM}, the LMTD, in the range of θ_1 and θ_2 of interest.

The advantage of having available an approximation to the LMTD has already been pointed out by Paterson [1] and it is suggested that in situations where an approximation is required, eq. (11) should be seriously considered.

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NOTATION

A	exchange area
Q	heat duty
U	overall heat transfer coefficient
θ	temperature difference

Subscripts

1, 2	ends of the exchanger
AM	arithmetic mean
CM1	approximation given in this work, eq. (7)
CM2	approximation given in this work, eq. (11)
GM	geometric mean
LM	logarithmic mean
PM	approximation of given by Paterson [1], eq. (1).
UM	approximation given by Underwood [2, 3], eq. (10)

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